Research Consulting Template

For Interested Parties

Month–Day-2019

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Abstract. This work will serve as a starting point for further and more in depth exploration.

1 Example Chapter: Introduction To Pressure in a liquid at depth:

First we introduce some physical constants, units, and unit conversions. Some conversions for pressure are

$$\begin{aligned} 1Pa &= 1\frac{N}{m^2} \\ 1.01325\times 10^5 Pa &= 1atm \\ &= P_0. \end{aligned} \tag{1}$$

Where Pa are Pascals, the SI unit for pressure, that are also in the units of Newtons N per meters squared m^2. Atmospheric pressure is atm, and the conversion to Pascals is above.

Some other physical constants are density ρ

$$\rho = \frac{m}{V} = \frac{\text{Mass}}{\text{Volume}}. \tag{2}$$

The density of pure water is

$$\rho_{pureWater} = 1000\frac{kg}{m^3}.$$

The density of Salt water is

$$\rho_{seaWater} = 1027\frac{kg}{m^3}. \tag{3}$$

This density varies little with depth, we will not worry about this for now.

The density of air at sea level at $20°C$ is

$$\rho_{air20°C} = 1.3\frac{kg}{m^3}. \tag{4}$$

The density of air at sea level at $0°C$ is

$$\rho_{air0°C} = 1.2\frac{kg}{m^3}.$$

1.1 Hydrostatic Equilibrium, Pressure in a Liquid at Rest.

Now we consider some physical relationships. In equilibrium the pressure in a static fluid decreases with height at a rate of

$$\frac{dP}{dz} = -\rho g.$$

Here P is pressure, ρ is the density of the fluid, and $g = 9.8\frac{m}{s^2}$ is gravitational acceleration. We integrate this and we get pressure as a function of z.

$$P(z) = P_1 + \rho g (z_1 - z). \tag{5}$$

Here P_1 is the known pressure at a surface interface, say atmospheric pressure at the surface of a body of water, and z_1 is the height from the surface interface. The z coordinate can be negative or positive, corresponding to a rise or fall in $P(z)$ pressure. This can be seen in figure 1.

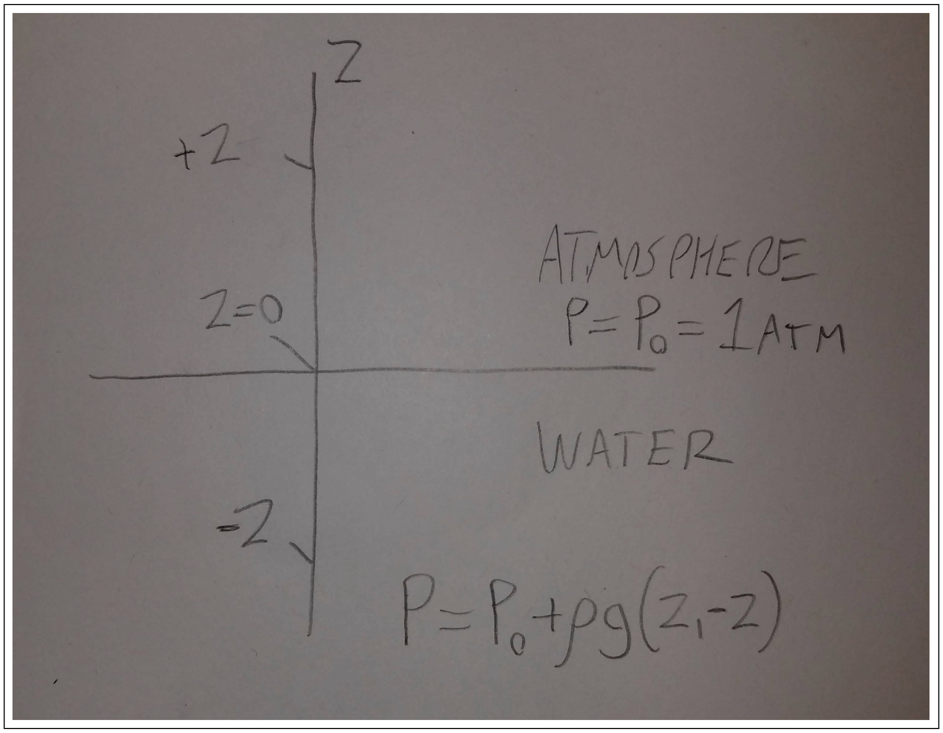

Fig. 1. Pressure at depth in a hydrostatic liquid, with a column of air above.

For example, at a depth of $10m$ into the ocean, the pressure, in units of Pascals, will be

$$\begin{aligned} P\,(10m) &= 1.01325 \times 10^5 Pa + \left(1027 \frac{kg}{m^3}\right)\left(9.8 \frac{m}{s^2}\right)(0 - (-10m)) \\ &= 1.01325 \times 10^5 Pa + 100646 \frac{kg}{m^3}\frac{m}{s^2} m \\ &= 1.01325 \times 10^5 Pa + 100646 \frac{N}{m^2} \\ &= 201971 Pa. \end{aligned} \tag{6}$$

In atms we have

$$201971 Pa \frac{1atm}{101325 Pa} = 1.99 atm.$$

1.2 Fluid Dynamics, Streamlines and Laminar Flow.

For fluid dynamics we will consider a liquid moving at a velocity in a tube. Initially we assume an incompressible fluid, with the flow along the stream lines being steady, irrotational and laminar. The physical setup is a stream tube with an area and velocity at both end, and can be seen in figure 2.

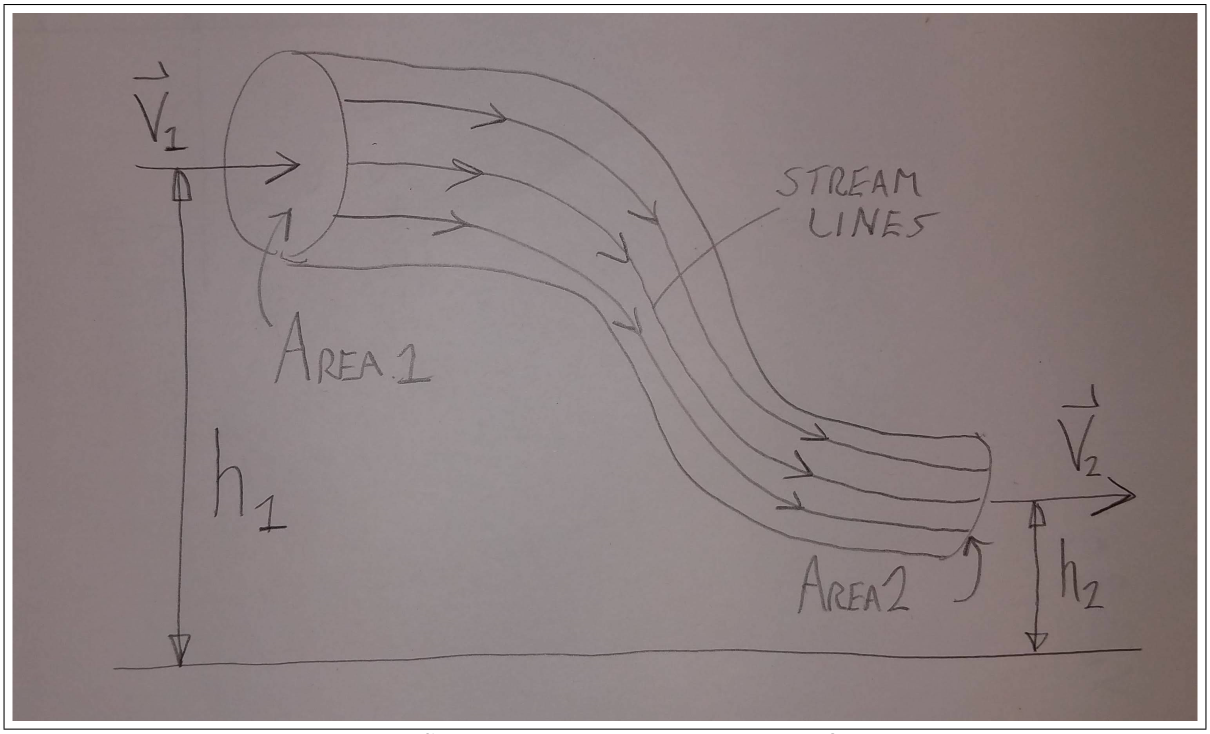

Fig. 2. Stream tube with liquid at steady flow.

We will use the two very important relations to describe the physics of this streamline, the Equation of Continuity and Bernoulli's Law. For a steady flow, the amount of mass m in the volume does not change, so the flow at any point along the stream tube is

$$\frac{dm}{dt} = \rho A v = \text{constant}.$$

We can set the Area and velocity on each end equal to each other, giving us the Equation of Continuity,

$$A_1 v_1 = A_2 v_2. \tag{7}$$

Here A_n is the area and v_n is the velocity at two opposite ends of a pipe.

Using the work energy theorem, to relate the work done by the net force to the change in kinetic energy gives us Bernoulli's Law,

$$P + \rho g z + \frac{1}{2}\rho v^2 = \text{constant along streamline}. \tag{8}$$

The first term P is the pressure in the pipe, and accounts for work done by internal forces. The second term, $\rho g z$, is the gravitational potential energy per unit volume and will depend on the definition of the starting values of height for different pressure situations. The third term is for the kinetic energy per unit volume. Next we consider the physical setup of interest.

2 Example Chapter: Finding solutions by matrix operations

Now that we have 4 equations, and four unknowns, we can use a matrix to find the solutions.

$$\begin{aligned} 2w - 3x + 2y + 5z &= 10 \\ 5w + x + 5y + z &= 2 \\ w + x + 3y - 7z &= 3 \\ -3w + 4x + 6y + z &= 7 \end{aligned}$$

To demonstrate the usefulness of a matrix, we use an example of solving equations. Our first two equations are

$$\begin{aligned} 3x + 2y &= 10 \\ x + 5y &= 2 \end{aligned} \tag{9}$$

Now we put these into a matrix. The coefficient matrix is

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$$

The variable matrix is

$$x = \begin{pmatrix} x \\ y \end{pmatrix}$$

The constant matrix is

$$B = \begin{pmatrix} 10 \\ 2 \end{pmatrix}$$

Writing this out gives us the equation

$$Ax = B. \tag{10}$$

The solution is found by taking the inverse of matrix A, and multiplying by matrix B.

$$\begin{aligned} A^{-1}Ax &= A^{-1}B \\ x &= A^{-1}B \end{aligned} \tag{11}$$

The inverse of A is

$$A^{-1} = \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}, inverse : \begin{pmatrix} \frac{5}{13} & -\frac{2}{13} \\ -\frac{1}{13} & \frac{3}{13} \end{pmatrix} \tag{12}$$

Now we multiply $A^{-1}B$

$$A^{-1}B = \begin{pmatrix} \frac{5}{13} & -\frac{2}{13} \\ -\frac{1}{13} & \frac{3}{13} \end{pmatrix} \begin{pmatrix} 10 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{46}{13} \\ -\frac{4}{13} \end{pmatrix} \tag{13}$$

Our result is

$$\begin{aligned} x &= A^{-1}B \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} \frac{5}{13} & -\frac{2}{13} \\ -\frac{1}{13} & \frac{3}{13} \end{pmatrix} \begin{pmatrix} 10 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{46}{13} \\ -\frac{4}{13} \end{pmatrix} \end{aligned} \tag{14}$$

3 Example Chapter: Behavior of the solution function

To get a understanding of the solution function, we use the concepts from Precalculus to analyze the behavior as we approach the vertical asymptote.

Lets look at the function,

$$y = \frac{x^2}{(x^2 - 1)}$$

A simple example of how the function works. this is seen in Fig 3.

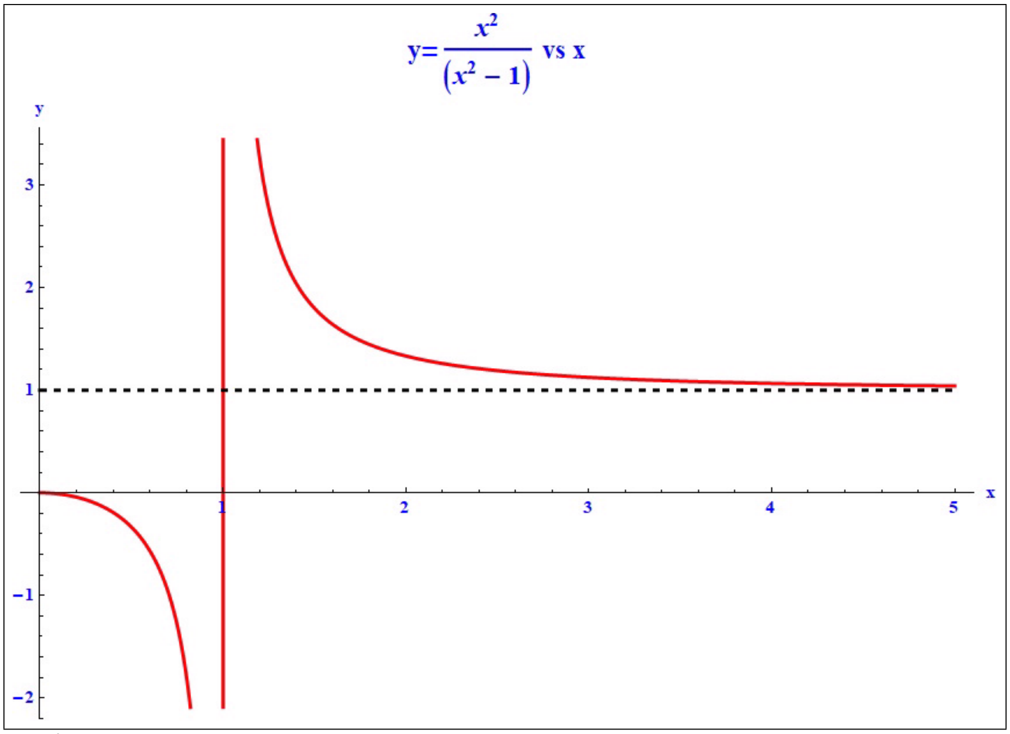

Figure 3. The function can not have negative solutions, and may have a larger solution, but will not be less than $y = 1$.

The red line represents the function. We see that when x is smaller than one, we get a negative solution going down to negative infinity, $-\infty$, as we approach $x = 1$. There is a vertical asymptote at $x = 1$. A vertical asymptote occurs when we have

$$y = \frac{(1)^2}{(1)^2 - 1} = \frac{1}{1-1} = \frac{1}{0}.$$

As x increases from $x = 1$, our solution comes down from positive infinity, $+\infty$, and the solution becomes smaller as x becomes larger. We also see that the solution can never become smaller than $y = 1$, as there is a horizontal asymptote at $y = 1$, and is found by dividing the numerator by the denominator.

$$y = \frac{x^2}{x^2} = 1$$

This tells us many things about the function, depending on the physical parameters.

4 Example Chapter: Table of Solutions for your problem.

To easily compare all of the important values, a table is constructed. Here we have a table 1 of values for Gold nanorods.

▾ PART #A13, MICROGOLD, 0.05MG/ML

Part #	Diameter (nm)	Length (nm)	Aspect Ratio	Peak SPR Wave (nm)	SPR OD (1cm)	Microgold Vol (nm3)	Microgold /mL	Wt. conc (ug/ml)	Wt. %	PPM	Molarity (pM)	SPR Molar Ext. (M-1cm-1)
A13-500	75	500	6.7	510	0.05	2.10E+06	1.24E+09	50	0.01%	50	2.06	2.43E+07
A13-1000	100	1000	10	510	0.05	7.59E+06	3.42E+08	50	0.01%	50	0.57	8.78E+07

▸ FUNCTIONALIZATIONS

Table 1: Table of values for Micro Gold Rods L=5000 nm D=75nm Aspect ratio 6.7 .

In the table above we see that....

5 Example Chapter: Conclusion

In conclusion, we have found that your idea is plausible......